COMPUTING MAXWELL EIGENVALUES IN 3d USING CONFORMING hp FEM

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Maxwell Equations Using Conforming FEM

We consider the following problem:
Find the Eigenvalues $\lambda = \omega^2$ corresponding to frequencies $\omega > 0$ such that $\exists(\boldsymbol{E}, \boldsymbol{H}) \neq 0$ satisfies

$$\operatorname{curl} \boldsymbol{E} - i\omega\mu\boldsymbol{H} = 0 \qquad \text{and} \qquad \operatorname{curl} \boldsymbol{H} + i\omega\varepsilon\boldsymbol{E} = 0 \qquad \text{in } \Omega,$$

with perfect conductor boundary conditions $\boldsymbol{E} \times \boldsymbol{n} = 0$ and $\boldsymbol{H} \cdot \boldsymbol{n} = 0$ on $\partial\Omega$. $\boldsymbol{E}$ belongs to $H_0(\mathrm{curl};\Omega)$.
The "electric" variational formulation:
Find the frequencies $\omega > 0$ such that

$$\exists \boldsymbol{E} \in H_0(\mathrm{curl};\Omega) \setminus \{0\} \text{ with } \int_\Omega {}^1\!/\!_\mu \operatorname{curl} \boldsymbol{E} \cdot \operatorname{curl} \boldsymbol{F} = \omega^2 \int_\Omega \varepsilon \boldsymbol{E} \cdot \boldsymbol{F} \text{ and } \operatorname{div} \varepsilon \boldsymbol{E} = 0 \quad \forall \boldsymbol{F} \in H_0(\mathrm{curl};\Omega).$$

[1] shows how conforming FEM can be used to solve Maxwell Eigenvalue problems. We choose $\varepsilon = 1 = \mu$.
The main point in the used discretization:
Find the frequencies $\omega > 0$ such that

$$\exists \boldsymbol{u} \in X_N \text{ with } \int_\Omega \operatorname{curl} \boldsymbol{u} \cdot \operatorname{curl} \boldsymbol{v} + \langle \boldsymbol{u}, \boldsymbol{v} \rangle_Y = \omega^2 \int_\Omega \boldsymbol{u} \cdot \boldsymbol{v} \quad \forall \boldsymbol{v} \in X_N := \{\boldsymbol{u} \in H_0(\mathrm{curl};\Omega) : \operatorname{div} \boldsymbol{u} \in L^2(\Omega)\}$$

is the bilinear form $\langle \boldsymbol{u}, \boldsymbol{v} \rangle_Y$:

$$\langle \boldsymbol{u}, \boldsymbol{v} \rangle_Y = s \int_\Omega \rho(\boldsymbol{x}) \operatorname{div} \boldsymbol{u} \operatorname{div} \boldsymbol{v}$$

with a properly chosen weight $\rho(\boldsymbol{x})$ and $s \in \mathbb{R}_+$. A good choice is $\rho(\boldsymbol{x}) = r^\alpha$ where r is the distance to a reentrant corner and $\alpha \geq 0$ in a range depending on the angle of the reentrant corner.

Conforming hp FEM in 3D

The software Concepts [2] used to compute the problem given above, is described: data structures, algorithms. Concepts is able to handle anisotropic approximation orders (p) in every element and anisotropic h refinements: The theoretical basics are given.

Results and Outlook

Exponential convergence for diffusion problems (the theory and the software take this as a basis) and Maxwell Eigenvalues for selected benchmark problems are shown.

References

[1] Martin Costabel and Monique Dauge, "Weighted regularization of Maxwell equations in polyhedral domains", *Numer. Math.* 93 (2), pp. 239–277 (2002).
[2] P. Frauenfelder and Ch. Lage, "Concepts—An Object Oriented Software Package for Partial Differential Equations", *Mathematical Modelling and Numerical Analysis* 36 (5), pp. 937–951 (2002).